2018

MATHEMATICS

Semester-I Examination (DODL)

Paper : MATC-1.1

(Real Analysis-I, Complex Analysis-I, Functional Analysis-I)

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

Write the answers to questions of each Block in separate books.

Block-I

(Real Analysis-I)

(Marks : 25)

Answer **Q.No.1** and any **two** questions from the rest:

1. Question (a) **or** (b):

   a) Define addition of two cardinal numbers with justification. Show that for any function f on $[a,b]$, $V(a, b; f)=0$ if f is a constant function on $[a, b]$. 3+2=5

[Turn over]

b) Show that a function f on the closed interval $[a,b]$ is of bounded variation on $[a,b]$ if its derivative exists and is bounded on $[a,b]$. 5

2. a) Show by an example that a bounded function f on a closed interval $[a,b]$ may not be a function of bounded variation on $[a,b]$. 4

b) Show that the variation function $F(x)=V_f(a,x)$ is absolutely continuous on $[a,b]$ if f is so. 6

3. a) If a function f is Riemann-Stieltjes integrable with respect to a bounded monotonic function α on the closed interval $[a,b]$, then show that, for every $\varepsilon > 0$, there exists a partition P of $[a,b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$. 5

b) Prove or disprove: Every bounded function f on a closed interval $[a,b]$ is Riemann-Stieltjes integrable for any monotonic increasing bounded function α on $[a,b]$. 5

4. a) State and prove the First Mean Value theorem for Riemann-Stieltjes Integrals. 6

b) Show that the set F of the real valued continuous functions from one interval I to another interval J has the power of the continuum. 4

Block - II

(Complex Analysis-I)

(Marks : 30)

Answer any **three** questions :

5. a) Show that if a function f is differentiable at a point z_0, then it is continuous there. Is the converse true? Justify your answer. 5

b) Let

$$f(z) = \frac{x^3 - y^3}{x^2 + y^2} + i\frac{x^3 + y^3}{x^2 + y^2}, z \neq 0$$

$$= 0, z = 0$$

Show that f satisfies CR equations at (0,0), but $f'(0)$ does not exist. 5

6. a) State Cauchy-Goursat theorem. Is the theorem true for any multiply-connected region? Show that

$$\int_{\gamma_1} f(z)\,dz = \int_{\gamma_2} f(z)\,dz$$

for a function f, analytic within a simply connected region G, and $\alpha,\ \beta \in G$ where γ_1 and γ_2 are any two paths lying in G, that joins α and β. 2+2+2=6

b) State and prove the Fundamental theorem of Algebra. 4

7. a) State and prove Cauchy's Integral formula for derivatives. 6

b) Evaluate the integral

$$\int_C \frac{e^z \sin z}{(z+4)(z-2)^2}$$

where, C is the circle $|z| = 3$. 4

8. a) Let $f(z) = u + iv$ be analytic in a domain D and $|f(z)|$ is equal to a constant in D. Show that $f(z)$ is constant in D. 4

b) State and prove Cauchy-Hadamard theorem for the power series

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n$$ 6

9. a) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \left[3+(-1)^n\right]^n z^n$$

Find the Laurent's expansion of

$$f(z) = \frac{1}{(z+1)(z+3)}$$

in $0 < |z+1| < 2$ 2+3=5

b) Show that

$$\cosh\left(z + \frac{1}{z}\right) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n}\right)$$

where $a_n = \frac{1}{2\pi} \int_0^{2\pi} \cos n\theta \cosh(2\cos\theta)\, d\theta$

5

Block - III.

(Functional Analysis-I)

(Marks : 25)

Answer **Q. No. 10** and any **two** from the rest:

10. Answer any **one** question: 5

a) State and prove Cantor's Intersection theorem.

b) Show that the space l_p is a complete metric space.

11. a) State and prove Banach Contraction Principle. 1+5=6

b) Show that every normed linear space may not be a Banach space. 4

12. a) Show that a normed linear space $X \neq \{0\}$ is a Banach space iff the unit sphere $S(0;1) = \{x \in X : \|x\| = 1\}$ is complete. 5

b) Define equivalent norms on a linear space. Prove that any two norms on a finite dimensional linear space X are equivalent. 1+4=5

13. a) Suppose the unit sphere $S(0;1) = \{x \in X : \|x\| = 1\}$ in a normed linear space X is compact. Show that X is finite dimensional. 5

b) Prove that every closed and bounded subset of a normed linear space X of finite dimension is compact. 5

2018

MATHEMATICS

Semester-I Examination (DODL)

Paper : MATC-1.2

(Ordinary Differential Equations, Partial Differential Equations)

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

Write the answers to questions of each Block in separate books.

Block-I

(Ordinary Differential Equations)

(Marks: 40)

Answer any **four** questions: $10\times4=40$

1. a) State Abel's identity for second order linear homogeneous differential equation. Hence find the Wronskian of $y_1(x)$ and $y_2(x)$ at the point $x=\frac{\pi}{4}$, if $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation

[Turn over]

$$(\cos x)y'' + (\sin x)y' - \left(1 + e^{-x^2}\right)y = 0, \ \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

with $y_1(0) = \sqrt{2},\ y_1'(0) = 1,\ y_2(0) = -\sqrt{2}, y_2'(0) = 2.$

b) Using variation of parameters, find a particular solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 64xe^{-x}.$$

1+4+5=10

2. a) Define fundamental set of solutions. If $\phi_1(x)$ is a solution of

$$L(y) = y'' + p_1(x)y' + p_2(x)y = 0$$

on an interval I and $\phi_1(x) \neq 0\ \forall x \in I$, then prove that a second solution $\phi_2(x)$ on I is given by

$$\phi_2(x) = \phi_1(x)\int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp\left[-\int_{x_0}^{x} p_1(t)\,dt\right] ds$$

b) Write the normal form of the linear second order differential equation $y'' + P(x)y' + Q(x)y = R$. Hence solve the following equation by reducing to normal form:

$$y'' - (2\tan x)y' - \left(a^2 + 1\right)y = e^x \sec x$$

1+4+1+4=10

3. a) Define Green's function for a second order non-homogeneous boundary value problem. Construct Green's function for the following boundary value problem and hence solve it:

$$\frac{d^2u}{dx^2} - u + 1 = 0, \quad 0 < x < 1$$

$$u(0) = 0, \; u(1) = 0$$

b) Examine whether a Green's function exists or not for the following boundary value problem:

$$y'' + y' = 0; \; y(0) = y(1), \; y'(0) = y'(1).$$

c) Show that the following differential equation is not a self adjoint equation and hence transform it into an equivalent self adjoint equation:

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$ 1+4+2+3=10

4. a) Prove that the second order homogeneous differential equation $a_0(t)x'' + a_1(t)x' + a_2(t)x = 0$ can be reduced to self adjoint form by multiplying throughout by the factor

$$h(t) = \frac{1}{a_0(t)} \exp\left(\int \frac{a_1(t)}{a_0(t)} dt\right).$$

b) State and prove Sturm separation theorem.

c) Find the eigen values and eigen functions of the differential equation

$$\frac{d^2y}{dx^2}+\lambda y=0,\ y(0)=0,\ y'(\pi)=0.$$

2+4+4=10

5. a) Using Grownwall's Lemma, prove that the solution of the following initial value problem is unique:

$$\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0 \text{ on } [x_0, x_0+h]$$

b) State Picard's existence and uniqueness theorem. Hence find the largest interval of existence of the solution of following initial value problem over the region R:

$$\frac{dy}{dx}=y^2,\ y(0)=2$$

Here R is the rectangle given by

$$R=\{(x,y):|x|\le a,\ |y-2|\le b,\ a>0,\ b>0\}.$$

1+4+1+4=10

6. a) State Fuch's theorem. Hence obtain the solution of the Legendre differential equation

$$(1-z^2)\frac{d^2w}{dz^2}-2z\frac{dw}{dz}+n(n+1)w=0$$

in the neighbourhood of $z=0$.

b) Define regular singularity and irregular singularity for a second order linear homogeneous equation. Use the method of Frobenius to find solutions of the differential equation

$$2z^2\frac{d^2w}{dz^2}-z\frac{dw}{dz}+(z-5)w=0$$

in some interval $0 < x < R$. 1+4+1+4=10

Block - II

(Partial Differential Equations)

(Marks : 40)

Answer any **four** questions: 10×4= 40

7. a) Prove that the general solution of the linear partial differential equation $Pp + Qq = R$ can be written in the form $F(u, v) = 0$, where F is an arbitrary function, and $u(x,y,z) = C_1$ and $v(x.y.z) = C_2$ form a solution of the equation

$$\frac{dx}{P(x,y,z)}=\frac{dy}{Q(x,y,z)}=\frac{dz}{R(x,y,z)}$$

b) State necessary and sufficient condition for compatibility of two partial differential equations
$f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$.

Show that the partial differential equations

$$xp - yq = x \text{ and } x^2p + q = xz.$$

are compatible and hence, find their solution.

4+1+2+3=10

8. a) Let u(x,*l*) be a function that satisfies the PDE

$$u_{xx} - u_{yy} = e^x + 6y,\ x \in \mathbb{R},\ y > 0$$

and the initial conditions

$$u(x,0) = \sin(x),\ u_y(x,0) = 0 \text{ for every } x \in \mathbb{R}$$

Here subscripts denote partial derivatives with respect to the variables indicated. Find the value of $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

b) Find by the method of characteristics, the integral surface of pq = xy which passes through the line z = x. y = 0. 5+5=10

9. a) Suppose that u(x, y) is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$. Prove that u attains its maximum on the boundary B.

b) Prove that the following partial differential equation is parabolic and find its canonical form. Hence find the general solution

$$x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$$

4+1+4+1=10

10. a) State the necessary and sufficient conditions for integrability of total differential equation $Pdx + Qdy + Rdz = 0$. Test the integrability of the following pfaffian differential equation and hence solve it.

$$(y-z)dx+(z-x)dy+(x-y)dz=0$$

b) Deduce D'Alembert's solution for vibration in an infinite string. 1+4+5=10

11. Obtain the solution of the Dirichlet's problem:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0<x<l, 0<y<1$$

subject to the boundary conditions

$$u(0,y)=0,\ u(l,y)=0,\ 0\le y\le 1,$$
and $u(x,0)=0,\ u(x,1)=x(l-x)$. 10

12. Solve the one-dimensional diffusion equation

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$$

in the region $0\le x\le \pi, t\ge 0$ subject to the conditions

i) T ramain finite as $t\to\infty$

ii) T=0, if $x=0$ and π for all t

iii) At $t = 0$, $T = \begin{cases} x, & 0 \le x \le \dfrac{\pi}{2} \\ \pi - x, & \dfrac{\pi}{2} \le x \le \pi \end{cases}$

10

2018

MATHEMATICS

Semester-I Examination (DODL)

Paper : MATC-1.3

(Mechanics-I, Abstract Algebra-I, Operations Research-I)

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

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Write the answers to questions of each Block in separate books.

Block-I

[Mechanics-I (Potential Theory)]

(Marks: 30)

Answer any **three** questions:

1. Give a physical interpretation of potential. Show that the second derivatives of the Newtonian potential U of a body at an external point exist and U satisfies Laplace's equation. 3+3+4=10

2. Prove that the potential U and the force components X, Y, Z of a volume distribution of matter of

[Turn over]

piecewise continuous density in the bounded volume V are continuous throughout the space. 5+5=10

3. State and prove the existence of a solution of first boundary value problem. Deduce Green's function of Dirichlet exterior problem. 3+3+4=10

4. State second boundary value problem. State and prove the existence of a solution of first boundary value problem. 3+2+5=10

5. Deduce Green's Second Identity. Use it to prove that every regular harmonic function can be represented as the sum of potentials due to surface distribution of matter on the surface S and double layer on S. 4+6=10

Block - II

(Abstract Algebra-I)

(Marks: 25)

6. Answer any **five** questions:

   a) If G is a group and S a G-set, then show that the left action of G induces a homomorphism from G onto A(S), where A(S) is the group of permutations of S. 5

   b) Define internal direct product of two groups G and H. Show that for any two groups A and B, $A \times B$ is abelian if and only if A and B are abelian. 2+3=5

c) Define a p-group where p is a prime. If G is a finite p-group with $|G| > 1$, then show that $|Z(G)| > 1$. Write down the class equation for S_3. 1+3+1=5

d) Let G be a finite group of order n such that a prime p divides n. Show that G has a subgroup of order p. 5

e) Define composition series of a group. Show that a subnormal series for a group is a composition series if and only if it has no proper refinement. 1+4=5

f) Let A, B, C, D be groups such that A and B are isomorphic and C and D are isomorphic. Show that $A \times C$ and $B \times D$ are isomorphic. Also show that $\mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_2$ are not isomorphic. 3+2=5

g) State Sylow's third theorem. Show that no group of order 108 is simple. 1+4=5

h) Define solvable group. Deduce a necessary and sufficient condition for a group to be solvable. 1+4=5

Block - III

(Operations Research-I)

(Marks: 25)

Answer Q. No. **7** and any **one** from the rest:

7. a) State the advantages of Revised Simplex method.

   b) Use the revised simplex method to solve the following linear programming problem:

   Maximize $Z = 2x_1 + x_2$

   subject to the constraint

   $3x_1 + 4x_2 \le 6,\ 6x_1 + x_2 \le 3$

   and $x_1, x_2 \ge 0$. 3+10=13

OR

   a) Define quardratic programming problem.

   b) Briefly describe Wolfe's algorithm for solving a quadratic programming problem.

   c) Use Wolfe's method to solve the quadratic programming problem:

   Maximize

   $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

   subject to the constraint

   $x_1 + 2x_2 \le 2$ and $x_1, x_2 \ge 0$.

   1+4+8=13

8. a) Briefly describe the importance of Integer Programming problem.

b) Classify different types of Integer Programming problems.

c) Solve the following Integer Linear Programming problem using Gomory's cutting plane method.

Maximize $Z = x_1 + x_2$

subject to the constraint

$3x_1 + 2x_2 \leq 5,\ x_2 \leq 2$

and $x_1, x_2 \geq 0$ and are integers.

2+2+8=12

9. a) What do you understand by the problem of sequencing? Discuss its applicability.

b) Explain the principal assumptions made while dealing with sequencing problems.

c) A book binder has one printing press, one binding machine and manuscripts of 7 different books. The time required for performing printing and binding operations for different books are shown below:

Book :	1	2	3	4	5	6	7
Printing time (hours):	20	90	80	20	120	15	65
Binding time (hours):	25	60	75	30	90	35	50

Decide the optimum sequence of processing of books in order to minimize the total time required to bring out all the books.

3+3+6=12

2018

MATHEMATICS

Semester-I Examination (DODL)

Paper : MATA-1.4

(Applied Stream)

(Mechanics of Solids, Non-linear Dynamics)

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

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Write the answers to questions of each Block in separate books.

Block-I

(Mechanics of Solids)

(Marks: 40)

Answer any **four** questions: 10×4=40

1. a) Define continuum. State and prove Cauchy's fundamental theorem.

   b) For the state of stress

$$\left(T_{ij}\right)=\begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

[Turn Over]

Determine principal stresses and principal directions. Also find the magnitude of the maximum stress. 2+4+4

2. a) Write down Lagrangian and Eulerian strain tensors for finite deformation of a continuous medium and show that there is no distinction between these two strain components in case of infinitesimal deformation.

b) Given the displacement field $x_1 = X_1+2X_3$, $x_2 = X_2-2X_3$, $x_3 = X_3-2X_1+2X_2$. Determine the deformation gradient, Green's deformation tensor and Lagrangian finite strain tensor.

5+5

3. a) Define Strain Quadric. Show that all principal strains are real.

b) Determine the principal direction and principal strains for

$$\left(E_{ij}\right)=\begin{bmatrix} e & e & e \\ e & e & e \\ e & e & e \end{bmatrix}.$$

2+4+4

4. a) State the necessary and sufficient condition for a given surface $F(x_1, x_2, x_3, t) = 0$ to be a boundary surface.

b) Find the condition that

$$\frac{x^2}{a^2}f_1(t) + \frac{y^2}{b^2}f_2(t) + \frac{z^2}{c^2}f_3(t) = 1$$ is a possible form of a boundary surface for an incompressible flow.

c) Show that equation of continuity in Lagrangian and Eulerian method are equivalent. 1+4+5

5. a) State generalized Hooke's law.

b) Deduce Beltrami-Michell compatibility equation for stress.

c) State fundamental boundary value problems in electrostatics. 2+5+3

6. a) What is plane stress? Explain how Airy's stress function can be used to determine the stress distribution in an elastic body in case of plane stress and in the absence of body forces.

b) Show that there are two possible plane waves propagating in an isotropic elastic medium.

5+5

Block-II

(Non-linear Dynamics)

(Marks: 40)

Answer any **four** questions: 10×4=40

7. a) Define non-autonomous system.

   b) State and prove the fundamental theorem for linear system.

   c) Solve the following linear coupled system and draw the phase portrait:

$$\dot{x}_1 = -x_1 - 3x_2$$
$$\dot{x}_2 = 2x_2$$

1+4+5

8. a) Define hyperbolic critical point. State Hartman-Grobman theorem.

   b) Show that (0, 0) is a critical point for the following system and discuss its stability:

$$\dot{x}(t) = x - 2y + y^2 \sin(x)$$
$$\dot{y}(t) = 2x - 2y - 3y \cos(y^2)$$

   c) Find the equilibrium points of the following Prey-Predator model and discuss their stability:

$$\dot{H}(t) = a_1H(t) - b_1P(t)H(t)$$
$$\dot{P}(t) = -a_2H(t) + b_2H(t)P(t)$$

2+4+4

9. a) Define asymptotic stability of an equilibrium point.

b) State Dulac's criterion.

c) Using Dulac's criterion with the weighting function $g = (N_1N_2)^{-1}$, show that the following system has no periodic orbits in the first quadrant $N_1, N_2 > 0$:

$$\dot{N}_1 = r_1N_1\left(1-\frac{N_1}{K_1}\right) - b_1N_1N_2$$

$$\dot{N}_2 = r_2N_2\left(1-\frac{N_2}{K_2}\right) - b_2N_1N_2.$$

2+2+6

10. a) Use the Liapunov function $V(x) = x_1^2 + x_2^2 + x_3^2$ to show that the origin is an asymptotically stable equilibrium point of the system

$$\dot{x}_1 = -x_2 - x_1x_2^2 + x_3^2 - x_1^3$$

$$\dot{x}_2 = x_1 + x_3^3 - x_2^3$$

$$\dot{x}_3 = -x_1x_3 - x_3x_1^2 - x_2x_3^2 - x_3^5$$

b) Consider the system

$$\dot{x} = -y + ax(x^2 + y^2)$$

$$\dot{y} = x + ay(x^2 + y^2)$$

where 'a' is a parameter. Show that the linearized system incorrectly predicts that the origin is a center for all values of a, whereas in fact the origin is a stable spiral if $a < 0$ and an unstable spiral if $a > 0$. 5+5

11. a) What do you mean by 'bifurcation' of a dynamical system?

b) Discuss transcritical bifurcation of the system $\dot{x} = \mu x - x^2, x \in \mathbb{R}$ with $\mu \in \mathbb{R}$ as a parameter.

c) Obtain the relation between 'a' and 'b', so that the system

$$\dot{x} = x(1 - x^2) - a(1 - e^{-bx})$$

undergoes a transcritical bifurcation at $x = 0$.

1+5+4

12. Consider the following system:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - h$$

Find the equilibrium points and check the stability of the system around the equilibrium points. Also find the critical value of h for a bifurcation to occur.

3+3+4

2018

MATHEMATICS

Semester-I Examination (DODL)

Paper : MATP-1.4

(Pure Stream)

(Differential Geometry-I, Topology-I)

Full Marks : 80 Time : 4 Hours

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Block-I

(Differential Geometry-I)

(Marks: 40)

Answer any **four** questions: 10×4=40

1. a) Find the arc length of the curve

$$\gamma(t) = (e^t \cos t, e^t \sin t).$$ 3

b) Define reparametrization of a unit speed curve. Show that reparametrization of a regular curve is regular. 7

[Turn Over]

2. a) Prove that the curvature of a regular curve is given by

$$\kappa = \frac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}\|^3}.$$ 6

b) Compute the torsion of the helix

$\gamma(\theta) = (a \cos \theta, a \sin \theta, b\theta)$. 4

3. Let $\kappa : (\alpha, \beta) \to \mathbb{R}$ be any smooth function. Show that there is a unit smooth curve $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ whose signed curvature is κ. 10

4. a) State and prove Wirtinger inequality. 5

b) State and prove four vertex theorem. 5

5. Show that a sphere is a smooth surface. 10

6. a) Show that a Möbius band is not orientable. 6

b) Show that transition maps of smooth surfaces are smooth. 4

Block-II

(Topology-I)

(Marks: 40)

Answer any **four** questions: 10×4=40

7. a) Show that arbitrary intersection of topologies on a set X always forms a topology on X. Is the result true for arbitrary union of topologies? Justify. 3+1

   b) Define neighbourhood of a point in a topological space. Show that a subset U of a topological space (X, τ) is open iff it is the neighbourhood of each of its points. 1+3

   c) Consider the real line with the usual topology and consider the sub sapce topology (Y, τ_Y), where $Y = [0, 1] \cup (2, 3)$. Is the set [0, 1] open in the subspace topology (Y, τ_Y)? Give justifications. 2

8. a) Define closure of a set in a topological space. Let A be a subset of a topological space (X, τ). Then show that $\bar{A} = A \cup A'$. 1+4

   b) Define homeomorphism between two topological spaces. Show that for any two topologies τ_1 and τ_2, on a set X, the identity map i: $(X, \tau_1) \rightarrow (X, \tau_2)$ is a homeomorphism if and only if $\tau_1 = \tau_2$. 1+3

c) Show that the two open intervals (a, b) and (0, 1) are homeomorphic in the real line with the usual topology. 1

9\. a) Show that if a function $f: X \to Y$ is continuous, then for any closed set V in the topological space Y, $f^{-1}(V)$ is closed in the topological space X. Show that any mapping on a set X with the discrete topology is continuous.

3+3

b) Show that for any subset A of a topological space (X, τ), $X \setminus \text{Int } A = \overline{X \setminus A}$. 3

c) Consider a continuous function $f: X \to \mathbb{R}$. Then prove that the set $G(f) = \{x \in X : f(x) \geq 0\}$ is a closed set in X. 1

10\. a) Define a Hausdorff space. Show that a subspace of a Hausdorff space is Hausdorff.

1+3

b) State Urysohn's Lemma. Show that for any two disjoint closed sets A and B in a topological space X, there exists a continuous function $g: X \to [a, b]$ such that $g(x) = a$, for $x \in A$ and $g(x) = b$, for $x \in B$. 1+2

c) If for any pair of disjoint closed sets A and B in a topological space X there is a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$, for $x \in A$, and $f(x) = 1$, for $x \in B$, then show that (X, τ) is normal. 3

11. a) Show that the real line with the co-finite topology is T_1 but not Hausdorff. 5

b) Let Y be a subspace of a topological space X. Show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. 4

c) Show that any singleton set in the real line with usual topology is closed. 1

12. a) State and prove a necessary and sufficient condition for a topological space (X, τ) to be normal. 1+4

b) State and prove the pasting lemma. 5
